

Exclusive $B \rightarrow K_1 \ell^+ \ell^-$ decay in model with single universal extra dimension

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Abstract

Decay rate and forward-backward asymmetries in $B \rightarrow K_1 \ell^+ \ell^-$, K_1 is the axial vector meson, are calculated in the universal extra dimension (UED) model. The dependence of these physical quantities on the compactification radius R , the only unknown parameter in UED model, is studied and it is shown that zero of forward-backward asymmetry is sensitive to the UED model, therefore they can be very useful tool to establish new physics predicted by the UED model. This work is briefly extended to $B \rightarrow K^* \ell^+ \ell^-$.

1 Introduction

The flavor-changing-neutral-current (FCNC) transitions $b \rightarrow s$ provide potentially stringent tests of standard model (SM) in flavor physics and are not allowed at tree level but are induced by the Glashow-Iliopoulos-Miani (GIM) amplitudes [1] at the loop level in the SM. In addition, these are also suppressed in SM due to their dependence on the weak mixing angles of the quark-flavor rotation matrix – the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2]. These two circumstances make the FCNC decays relatively rare and

hence important for the study of physics beyond the SM commonly known as new physics.

The experimental observation of inclusive [3] and exclusive [4] decays, $B \rightarrow X_s \gamma$ and $B \rightarrow K^* \gamma$, has prompted a lot of theoretical interest on rare B meson decays. Though the inclusive decays are theoretically better understood but are extremely difficult to be measured in a hadron machine, such as the LHC, which is the only collider, except for a Super- B factory, that could provide enough luminosity for the precise study of the decay distribution of such rare processes. In contrast, the exclusive decays are easy to detect experimentally but are challenging to calculate theoretically and the difficulty lies in describing the hadronic structure, which provides the main uncertainty in the predictions of exclusive rare decays. In exclusive $B \rightarrow K, K^*$ decays the long-distance effects in the meson transition amplitude of the effective Hamiltonian are encoded in the meson transition form factors which are the scalar functions of the square of momentum transfer and are model dependent quantities. Many exclusive $B \rightarrow K (K^*) \ell^+ \ell^-$ [10, 11, 12], $B \rightarrow \phi \ell^+ \ell^-$ [14], $B \rightarrow \gamma \ell^+ \ell^-$ [13], $B \rightarrow \ell^+ \ell^-$ [15] processes based on $b \rightarrow s (d) \ell^+ \ell^-$ have been studied in literature and many frameworks have been applied to the description of meson transition form factors: like constituent quark models, QCD sum rules, lattice QCD, approaches based on heavy quark symmetry and analytical constraints.

Rare B decay modes also provide important ways to look for physics beyond the SM. There are various extensions of the SM in the literature, but the models with extra dimensions are of viable interest as they provide a unified framework for gravity and other interactions. In this way they give some hints on the hierarchy problem and a connection with string theory. Among different models of extra dimensions, which differ from one another depending on the number of extra dimensions, the most interesting are the scenario with universal extra dimensions. In these UED models all the SM fields are allowed to propagate in the extra dimensions and compactification of extra dimension leads to the appearance of Kaluza-Klein (KK) partners of the SM fields in the four dimensional description of higher dimensional theory, together with KK modes without corresponding SM partners. The Appelquist, Cheng and Dobrescu (ACD) model [16] with one universal extra dimension (UED) is very attractive because it has only one free parameter with respect to the SM and that is the inverse of compactification radius R [17].

By analyzing the signature of extra dimensions in the different processes,

one can get bounds to the size of extra dimensions which are different in different models. These bounds are accessible for the processes already known at the particle accelerators or within the reach of planned future facilities. In case of UED these bounds are more severe and constraints from Tevatron run I allow to put the bound $1/R \geq 300$ GeV [17].

Rare B decays can also be used to constraint the ACD scenario and in this regard Buras and collaborators have already done some work. In addition to the effective Hamiltonian they have calculated for $b - s$ decays and also investigated the impact of UED on the $B^0 - \bar{B}^0$ mixing as well as on the CKM unitarity triangle [18, 19, 20]. Due to the availability of precise data on the decays $B \rightarrow K(K^*)\ell^+\ell^-$, Colangelo et al. have studied these decays in ACD model by calculating the branching ratio and forward-backward asymmetry for the decay $B \rightarrow K(K^*)\ell^+\ell^-$. We will study the rare semileptonic decay modes, $B \rightarrow K_1\ell^+\ell^-$ on the same footing as $B \rightarrow K^*\ell^+\ell^-$ because both are induced by the same quark level transitions, i.e. $b \rightarrow s\ell^+\ell^-$. We compare results of forward backward asymmetry for $B \rightarrow K^*\ell^+\ell^-$ using our form factors with those obtained by Colangelo *et al* [17]. The comparison shows clear distinction as shown in Fig 4. These decays may provide us step forward towards the study of existence of new physics beyond the SM and therefore deserve serious attention, both theoretically and experimentally.

The paper is organized as follows. In Section 2 we will briefly introduce the ACD model. Section 3 deals with the study of effective Hamiltonian and the corresponding matrix elements for $B \rightarrow K_1\ell^+\ell^-$ decay. Now the new physics manifest in these decays in two different ways, either through new operators in the effective Hamiltonian which are absent in the SM or through new contributions to the Wilson coefficients [14]. In ACD no new operator appears at tree level and therefore the new physics comes only through the Wilson coefficients which are calculated in literature [19, 20] and we will summarize them in the same section. Finally, in Section 4 we will calculate the decay rate and forward-backward asymmetry and summarize our results.

2 ACD Model

In our usual universe we have 3 spatial +1 temporal dimensions and if an extra dimension exists and is compactified, fields living in all dimensions would manifest themselves in the 3+1 space by the appearance of Kaluza-Klein ex-

citations. The most pertinent question is whether ordinary fields propagate or not in all extra dimensions. One obvious possibility is the propagation of gravity in whole ordinary plus extra dimensional universe, the “bulk”. Contrary to this there are the models with universal extra dimensions (UED) in which all the fields propagate in all available dimensions [16] and Appelquist, Cheng and Dobrescu model belongs to one of UED scenarios [17]

This model is the minimal extension of the SM in $4 + \delta$ dimensions, and in literature a simple case $\delta = 1$ is considered [17]. The topology for this extra dimension is orbifold S^1/Z_2 , and the coordinate $x_5 = y$ runs from 0 to $2\pi R$, where R is the compactification radius. The Kaluza-Klein (KK) mode expansion of the fields are determined from the boundary conditions at two fixed points $y = 0$ and $y = \pi R$ on the orbifold. Under parity transformation $P_5 : y \rightarrow -y$ the fields may be even or odd. Even fields have their correspondent in the 4 dimensional SM and their zero mode in the KK mode expansion can be interpreted as the ordinary SM field. The odd fields do not have their correspondent in the SM and therefore do not have zero mode in the KK expansion.

The significant features of the ACD model are:

- i) the compactification radius R is the only free parameter with respect to SM
- ii) no tree level contribution of KK modes in low energy processes (at scale $\mu \ll 1/R$) and no production of single KK excitation in ordinary particle interactions is a consequence of conservation of KK parity.

The detailed description of ACD model is provided in [19]; here we summarize main features of its construction from [17].

Gauge group

As ACD model is the minimal extension of SM therefore the gauge bosons associated with the gauge group $SU(2)_L \times U(1)_Y$ are W_i^a ($a = 1, 2, 3, i = 0, 1, 2, 3, 5$) and B_i , and the gauge couplings are $\hat{g}_2 = g_2\sqrt{2\pi R}$ and $\hat{g}' = g'\sqrt{2\pi R}$ (the hat on the coupling constant refers to the extra dimension). The charged bosons are $W_i^\pm = \frac{1}{\sqrt{2}}(W_i^1 \mp W_i^2)$ and the mixing of W_i^3 and B_i give rise to the fields Z_i and A_i as they do in the SM. The relations for the mixing angles are:

$$c_W = \cos \theta_W = \frac{\hat{g}_2}{\sqrt{\hat{g}_2^2 + \hat{g}'^2}} \quad c_W = \sin \theta_W = \frac{\hat{g}'}{\sqrt{\hat{g}_2^2 + \hat{g}'^2}} \quad (1)$$

The Weingberg angle remains the same as in the SM, due to the relationship between five and four dimensional constants. The gluons which are the gauge bosons associated to $SU(3)_C$ are $G_i^a(x, y)$ ($a = 1, \dots, 8$).

Higgs sector and mixing between Higgs fields and gauge bosons

The Higgs doublet can be written as:

$$\phi = \begin{pmatrix} i\chi^+ \\ \frac{1}{\sqrt{2}}(\psi - i\chi^3) \end{pmatrix} \quad (2)$$

with $\chi^\pm = \frac{1}{\sqrt{2}}(\chi^1 \mp i\chi^2)$. Now only field ψ has a zero mode, and we assign vacuum expectation value \hat{v} to such mode, so that $\psi \rightarrow \hat{v} + H$. H is the SM Higgs field, and the relation between expectation values in five and four dimension is: $\hat{v} = v/\sqrt{2\pi R}$.

The Goldstone fields $G_{(n)}^0$, $G_{(n)}^\pm$ arises due to the mixing of charged $W_{5(n)}^\pm$ and $\chi_{(n)}^\pm$, as well as neutral fields $Z_{5(n)}$. These Goldstone modes are then used to give masses to the $W_{(n)}^{\pm\mu}$ and $Z_{(n)}^\mu$, and $a_{(n)}^0$, $a_{(n)}^\pm$, new physical scalars.

Yukawa terms

In SM, Yukawa coupling of the Higgs field to the fermion provides the fermion mass terms. The diagonalization of such terms leads to the introduction of the CKM matrix. In order to have chiral fermions in ACD model, the left and right-handed components of the given spinor cannot be simultaneously even under P_5 . This makes the ACD model to be the minimal flavor violation model, since there are no new operators beyond those present in the SM and no new phase beyond the CKM phase and the unitarity triangle remains the same as in SM [19]. In order to have 4-d mass eigenstates of higher KK levels, a further mixing is introduced among the left-handed doublet and right-handed singlet of each flavor f . The mixing angle is such that $\tan(2\alpha_{f(n)}) = \frac{m_f}{n/R}$ ($n \geq 1$) giving mass $m_{f(n)} = \sqrt{m_f^2 + \frac{n^2}{R^2}}$, so that it is negligible for all flavors except the top [17].

Integrating over the fifth-dimension y gives the four-dimensional Lagrangian:

$$\mathcal{L}_4(x) = \int_0^{2\pi R} \mathcal{L}_5(x, y) \quad (3)$$

which describes: (i) zero modes corresponding to the SM fields, (ii) their massive KK excitations, (iii) KK excitations without zero modes which do not corresponds to any field in SM. Feynman rules used in the further calculation are given in Ref. [19].

3 Effective Hamiltonian

At quark level the decay $B \rightarrow K_1 \ell^+ \ell^-$ is same like $B \rightarrow K^* \ell^+ \ell^-$ as discussed by Ali *et al.* [10], i.e. $b \rightarrow s \ell^+ \ell^-$ and it can be described by effective Hamiltonian obtained by integrating out the top quark and W^\pm bosons

$$H_{eff} = -4 \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \quad (4)$$

where O_i 's are four local quark operators and \dot{C}_i are Wilson co-efficients calculated in Naive dimensional regularization (NDR) scheme [21].

One can write the above Hamiltonian in the following free quark decay amplitude

$$\mathcal{M}(b \rightarrow s \ell^+ \ell^-) = \frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{ts}^* \left\{ \begin{array}{l} C_9^{eff} [\bar{s} \gamma_\mu L b] [\bar{\ell} \gamma^\mu \ell] \\ + C_{10} [\bar{s} \gamma_\mu L b] [\bar{\ell} \gamma^\mu \gamma^5 \ell] \\ - 2 \hat{m}_b C_7^{eff} [\bar{s} i \sigma_{\mu\nu} \frac{\hat{q}^\nu}{\hat{s}} R b] [\bar{\ell} \gamma^\mu \ell] \end{array} \right\} \quad (5)$$

with $L/R \equiv \frac{(1 \mp \gamma_5)}{2}$, $s = q^2$ which is just the momentum transfer from heavy to light meson. The amplitude given in Eq. (5) contains long distance effects encoded in the form factors and short distance effects that are hidden in Wilson coefficients. These Wilson coefficients have been computed at next-to-next leading order (NNLO) in the SM [22]. Specifically for exclusive decays, the effective coefficient C_9^{eff} can be written as

$$C_9^{eff} = C_9 + Y(\hat{s}) \quad (6)$$

where the perturbatively calculated result of $Y(\hat{s})$ is [21]

$$Y_{\text{pert}}(\hat{s}) = \begin{aligned} & g(\hat{m}_c, \hat{s}) (3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \\ & - \frac{1}{2} g(1, \hat{s}) (4C_3 + 4C_4 + 3C_5 + C_6) \\ & - \frac{1}{2} g(0, \hat{s}) (C_3 + 3C_4) + \frac{2}{9} (3C_3 + C_4 + 3C_5 + C_6). \end{aligned} \quad (7)$$

Here the hat denote the normalization in term of B meson mass. For the explicit expressions of g 's and numerical values of the Wilson coefficients appearing in Eq. (7) we refer to [21].

In ACD model the new physics comes through the Wilson coefficients. Buras et al. have computed the above coefficients at NLO in ACD model

including the effects of KK modes [19, 20]; we use these results to study $B \rightarrow K_1 \ell^+ \ell^-$ decay. As it has already been mentioned that ACD model is the minimal extension of SM with only one extra dimension and it has no extra operator other than the SM, therefore, the whole contribution from all the KK states is in the Wilson coefficients, i.e. now they depend on the additional ACD parameter, the inverse of compactification radius R . At large value of $1/R$ the SM phenomenology should be recovered, since the new states, being more and more massive, decoupled from the low-energy theory. Our objective is to calculate the decay rate and forward-backward asymmetry for $B \rightarrow K_1 \ell^+ \ell^-$ using the lower bound on $1/R$ provided by Colangelo et al. for $B \rightarrow K^* \ell^+ \ell^-$ decay [17].

In ACD model, the Wilson coefficients are modified and they contain the contribution from new particles which are not present in the SM and comes as an intermediate state in penguin and box diagrams. Thus, these coefficients can be expressed in terms of the functions $F(x_t, 1/R)$, $x_t = \frac{m_t^2}{M_W^2}$, which generalize the corresponding SM function $F_0(x_t)$ according to:

$$F(x_t, 1/R) = F_0(x_t) + \sum_{n=1}^{\infty} F_n(x_t, x_n) \quad (8)$$

with $x_n = \frac{m_n^2}{M_W^2}$ and $m_n = \frac{n}{R}$ [17]. The relevant diagrams are Z^0 penguins, γ penguins, gluon penguins, γ magnetic penguins, Chromomagnetic penguins and the corresponding functions are $C(x_t, 1/R)$, $D(x_t, 1/R)$, $E(x_t, 1/R)$, $D'(x_t, 1/R)$ and $E'(x_t, 1/R)$ respectively. These functions can be found in [19, 20] but to make the paper self contained, we collect here the formulae needed for our analysis.

• C_7

In place of C_7 , one defines an effective coefficient $C_7^{(0)eff}$ which is renormalization scheme independent [23]:

$$C_7^{(0)eff}(\mu_b) = \eta^{\frac{16}{23}} C_7^{(0)}(\mu_w) + \frac{8}{3} (\eta^{\frac{14}{23}} - \eta^{\frac{16}{23}}) C_8^{(0)}(\mu_w) + C_2^{(0)}(\mu_w) \sum_{i=1}^8 h_i \eta^{\alpha_i} \quad (9)$$

where $\eta = \frac{\alpha_s(\mu_w)}{\alpha_s(\mu_b)}$, and

$$C_2^{(0)}(\mu_w) = 1, \quad C_7^{(0)}(\mu_w) = -\frac{1}{2} D'(x_t, \frac{1}{R}), \quad C_8^{(0)}(\mu_w) = -\frac{1}{2} E'(x_t, \frac{1}{R}); \quad (10)$$

the superscript (0) stays for leading log approximation. Furthermore:

$$\begin{aligned}
\alpha_1 &= \frac{14}{23} & \alpha_2 &= \frac{16}{23} & \alpha_3 &= \frac{6}{23} & \alpha_4 &= -\frac{12}{23} \\
\alpha_5 &= 0.4086 & \alpha_6 &= -0.4230 & \alpha_7 &= -0.8994 & \alpha_8 &= -0.1456 \\
h_1 &= 2.996 & h_2 &= -1.0880 & h_3 &= -\frac{3}{7} & h_4 &= -\frac{1}{14} \\
h_5 &= -0.649 & h_6 &= -0.0380 & h_7 &= -0.0185 & h_8 &= -0.0057. \quad (11)
\end{aligned}$$

The functions D' and E' are given by eq. (11) with

$$D'_0(x_t) = -\frac{(8x_t^3 + 5x_t^2 - 7x_t)}{12(1-x_t)^3} + \frac{x_t^2(2-3x_t)}{2(1-x_t)^4} \ln x_t \quad (12)$$

$$E'_0(x_t) = -\frac{x_t(x_t^2 - 5x_t - 2)}{4(1-x_t)^3} + \frac{3x_t^2}{2(1-x_t)^4} \ln x_t \quad (13)$$

$$\begin{aligned}
D'_n(x_t, x_n) &= \frac{x_t(-37 + 44x_t + 17x_t^2 + 6x_n^2(10 - 9x_t + 3x_t^2) - 3x_n(21 - 54x_t + 17x_t^2))}{36(x_t - 1)^3} \\
&+ \frac{x_n(2 - 7x_n + 3x_n^2)}{6} \ln \frac{x_n}{1 + x_n} \\
&- \frac{(-2 + x_n + 3x_t)(x_t + 3x_t^2 + x_n^2(3 + x_t) - x_n)(1 + (-10 + x_t)x_t)}{6(x_t - 1)^4} \ln \frac{x_n + x_t}{1 + x_n} \quad (14)
\end{aligned}$$

$$\begin{aligned}
E'_n(x_t, x_n) &= \frac{x_t(-17 - 8x_t + x_t^2 + 3x_n(21 - 6x_t + x_t^2) - 6x_n^2(10 - 9x_t + 3x_t^2))}{12(x_t - 1)^3} \\
&+ -\frac{1}{2}x_n(1 + x_n)(-1 + 3x_n) \ln \frac{x_n}{1 + x_n} \\
&+ \frac{(1 + x_n)(x_t + 3x_t^2 + x_n^2(3 + x_t) - x_n(1 + (-10 + x_t)x_t))}{2(x_t - 1)^4} \ln \frac{x_n + x_t}{1 + x_n} \quad (15)
\end{aligned}$$

Following [19] one gets the expressions for the sum over n :

$$\begin{aligned}
\sum_{n=1}^{\infty} D'_n(x_t, x_n) = & -\frac{x_t(-37 + x_t(44 + 17x_t))}{72(x_t - 1)^3} \\
& + \frac{\pi M_w R}{2} \left[\int_0^1 dy \frac{2y^{\frac{1}{2}} + 7y^{\frac{3}{2}} + 3y^{\frac{5}{2}}}{6} \right] \coth(\pi M_w R \sqrt{y}) \\
& + \frac{(-2 + x_t)x_t(1 + 3x_t)}{6(x_t - 1)^4} J(R, -\frac{1}{2}) \\
& - \frac{1}{6(x_t - 1)^4} [x_t(1 + 3x_t) - (-2 + 3x_t)(1 + (-10 + x_t)x_t)] J(R, \frac{1}{2}) \\
& + \frac{1}{6(x_t - 1)^4} [(-2 + 3x_t)(3 + x_t) - (1 + (-10 + x_t)x_t)] J(R, \frac{3}{2}) \\
& - \frac{(3 + x_t)}{6(x_t - 1)^4} J(R, \frac{5}{2})] \tag{16}
\end{aligned}$$

$$\begin{aligned}
\sum_{n=1}^{\infty} E'_n(x_t, x_n) = & -\frac{x_t(-17 + (-8 + x_t)x_t)}{24(x_t - 1)^3} \\
& + \frac{\pi M_w R}{2} \left[\int_0^1 dy (y^{\frac{1}{2}} + 2y^{\frac{3}{2}} - 3y^{\frac{5}{2}}) \coth(\pi M_w R \sqrt{y}) \right] \\
& - \frac{x_t(1 + 3x_t)}{(x_t - 1)^4} J(R, -\frac{1}{2}) \\
& + \frac{1}{(x_t - 1)^4} [x_t(1 + 3x_t) - (1 + (-10 + x_t)x_t)] J(R, \frac{1}{2}) \\
& - \frac{1}{(x_t - 1)^4} [(3 + x_t) - (1 + (-10 + x_t)x_t)] J(R, \frac{3}{2}) \\
& + \frac{(3 + x_t)}{(x_t - 1)^4} J(R, \frac{5}{2})] \tag{17}
\end{aligned}$$

where

$$J(R, \alpha) = \int_0^1 dy y^\alpha [\coth(\pi M_w R \sqrt{y}) - x_t^{1+\alpha} \coth(\pi m_t R \sqrt{y})]. \tag{18}$$

• C_9

In the ACD model and in the NDR scheme one has

$$C_9(\mu) = P_0^{NDR} + \frac{Y(x_t, \frac{1}{R})}{\sin^2 \theta_W} - 4Z(x_t, \frac{1}{R}) + P_E E(x_t, \frac{1}{R}) \tag{19}$$

where $P_0^{NDR} = 2.60 \pm 0.25[20]$ and the last term is numerically negligible. Besides

$$\begin{aligned} Y(x_t, \frac{1}{R}) &= Y_0(x_t) + \sum_{n=1}^{\infty} C_n(x_t, x_n) \\ Z(x_t, \frac{1}{R}) &= Z_0(x_t) + \sum_{n=1}^{\infty} C_n(x_t, x_n) \end{aligned} \quad (20)$$

with

$$\begin{aligned} Y_0(x_t) &= \frac{x_t}{8} \left[\frac{x_t - 4}{x_t - 1} + \frac{3x_t}{(x_t - 1)^2} \ln x_t \right] \\ Z_0(x_t) &= \frac{18x_t^4 - 163x_t^3 + 259x_t^2 - 108x_t}{144(x_t - 1)^3} \\ &\quad + \left[\frac{32x_t^4 - 38x_t^3 + 15x_t^2 - 18x_t}{72(x_t - 1)^4} - \frac{1}{9} \right] \ln x_t \end{aligned} \quad (21)$$

$$C_n(x_t, x_n) = \frac{x_t}{8(x_t - 1)^2} [x_t^2 - 8x_t + 7 + (3 + 3x_t + 7x_n - x_t x_n) \ln \frac{x_t + x_n}{1 + x_n}] \quad (22)$$

and

$$\sum_{n=1}^{\infty} C_n(x_t, x_n) = \frac{x_t(7 - x_t)}{16(x_t - 1)} - \frac{\pi M_w R x_t}{16(x_t - 1)^2} [3(1 + x_t)J(R, -\frac{1}{2}) + (x_t - 7)J(R, \frac{1}{2})] \quad (23)$$

• C_{10}

C_{10} is μ independent and is given by

$$C_{10} = -\frac{Y(x_t, \frac{1}{R})}{\sin^2 \theta_w}. \quad (24)$$

The normalization scale is fixed to $\mu = \mu_b \simeq 5 \text{ GeV}$.

Wilson coefficients give the short distance effects where as the long distance effects involve the matrix elements of the operators in Eq. (5) between the B and K_1 mesons. Using standard parameterization in terms of the form

factors we have [24]:

$$\begin{aligned}
\langle K_1(k, \varepsilon) | V_\mu | B(p) \rangle &= i\varepsilon_\mu^* (M_B + M_{K_1}) V_1(s) \\
&\quad - (p + k)_\mu (\varepsilon^* \cdot q) \frac{V_2(s)}{M_B + M_{K_1}} \\
&\quad - q_\mu (\varepsilon \cdot q) \frac{2M_{K_1}}{s} [V_3(s) - V_0(s)] \quad (25)
\end{aligned}$$

$$\langle K_1(k, \varepsilon) | A_\mu | B(p) \rangle = \frac{2i\epsilon_{\mu\nu\alpha\beta}}{M_B + M_{K_1}} \varepsilon^{*\nu} p^\alpha k^\beta A(s) \quad (26)$$

where $V_\mu = \bar{s}\gamma_\mu b$ and $A_\mu = \bar{s}\gamma_\mu\gamma_5 b$ are the vector and axial vector currents respectively and ε_μ^* is the polarization vector for the final state axial vector meson.

The relationship between different form factors which also ensures that there is no kinematical singularity in the matrix element at $s = 0$ is

$$V_3(s) = \frac{M_B + M_{K_1}}{2M_{K_1}} V_1(s) - \frac{M_B - M_{K_1}}{2M_{K_1}} V_2(s) \quad (27)$$

$$V_3(0) = V_0(0). \quad (28)$$

In addition to the above form factors there are also some penguin form factors which are:

$$\begin{aligned}
\langle K_1(k, \varepsilon) | \bar{s}i\sigma_{\mu\nu}q^\nu b | B(p) \rangle &= \left[(M_B^2 - M_{K_1}^2) \varepsilon_\mu - (\varepsilon \cdot q)(p + k)_\mu \right] F_2(s) \\
&\quad + (\varepsilon^* \cdot q) \left[q_\mu - \frac{s}{M_B^2 - M_{K_1}^2} (p + k)_\mu \right] F_3(s) \quad (29)
\end{aligned}$$

$$\langle K_1(k, \varepsilon) | \bar{s}i\sigma_{\mu\nu}q^\nu \gamma_5 b | B(p) \rangle = -i\epsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} p^\alpha k^\beta F_1(s) \quad (30)$$

with $F_1(0) = 2F_2(0)$.

Form factors are the non-perturbative quantities and are the scalar function of the square of momentum transfer. Different models are used to calculate these form factors. The form factors we use here in the analysis of physical variables like decay rate and forward-backward asymmetry have been calculated using Ward identities. The detailed calculation and their

expressions are given in ref. [24] and can be summarized as:

$$\begin{aligned}
A(s) &= \frac{A(0)}{(1 - s/M_B^2)(1 - s/M_B'^2)} \\
V_1(s) &= \frac{V_1(0)}{\left(1 - s/M_{B_A}^2\right)\left(1 - s/M_{B_A}'^2\right)} \left(1 - \frac{s}{M_B^2 - M_{K_1}^2}\right) \\
V_2(s) &= \frac{\tilde{V}_2(0)}{\left(1 - s/M_{B_A}^2\right)\left(1 - s/M_{B_A}'^2\right)} - \frac{2M_{K_1}}{M_B - M_{K_1}} \frac{V_0(0)}{(1 - s/M_B^2)(1 - s/M_B'^2)}
\end{aligned} \tag{31}$$

with

$$\begin{aligned}
A(0) &= -(0.52 \pm 0.05) \\
V_1(0) &= -(0.24 \pm 0.02) \\
\tilde{V}_2(0) &= -(0.39 \pm 0.03)
\end{aligned} \tag{32}$$

The corresponding values for $B \rightarrow K^*$ form factors at $s = 0$ are given by

$$\begin{aligned}
V(0) &= (0.29 \pm 0.04) \\
A_1(0) &= (0.23 \pm 0.03) \\
\tilde{A}_2(0) &= (0.33 \pm 0.05)
\end{aligned} \tag{33}$$

4 Decay Distribution and Forward-Backward Asymmetry

In this section we define the decay rate distribution which we shall use for the phenomenological analysis. Following the notation from ref.[10] we can write from Eq. (5)

$$\mathcal{M} = \frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{ts}^* m_B [\mathcal{T}_\mu^1 (\bar{l} \gamma^\mu l) + \mathcal{T}_\mu^2 (\bar{l} \gamma^\mu \gamma^5 l)] \tag{34}$$

where

$$\mathcal{T}_\mu^1 = A(\hat{s}) \varepsilon_{\mu\rho\alpha\beta} \epsilon^{*\rho} \hat{p}_B^\alpha \hat{p}_{K_1}^\beta - iB(\hat{s}) \epsilon_\mu^* + iC(\hat{s}) (\epsilon^* \cdot \hat{p}_B) \hat{p}_{h\mu} + iD(\hat{s}) (\epsilon^* \cdot \hat{p}_B) \hat{q}_\mu \tag{35}$$

$$\mathcal{T}_\mu^2 = E(\hat{s}) \varepsilon_{\mu\rho\alpha\beta} \epsilon^{*\rho} \hat{p}_B^\alpha \hat{p}_{K_1}^\beta - iF(\hat{s}) \epsilon_\mu^* + iG(\hat{s}) (\epsilon^* \cdot \hat{p}_B) \hat{p}_{h\mu} + iH(\hat{s}) (\epsilon^* \cdot \hat{p}_B) \hat{q}_\mu \tag{36}$$

The definition of different momenta involved are defined in reference[10], where the auxiliary functions are

$$\begin{aligned}
A(\hat{s}) &= -\frac{2A(\hat{s})}{1 + \hat{M}_{K_1}} C_9^{eff}(\hat{s}) + \frac{2\hat{m}_b}{\hat{s}} C_7^{eff} F_1(\hat{s}) \\
B(\hat{s}) &= \left(1 + \hat{M}_{K_1}\right) \left[C_9^{eff}(\hat{s}) V_1(\hat{s}) + \frac{2\hat{m}_b}{\hat{s}} C_7^{eff} \left(1 - \hat{M}_{K_1}\right) \right] \\
C(\hat{s}) &= \frac{1}{\left(1 - \hat{M}_{K_1}^2\right)} \left\{ C_9^{eff}(\hat{s}) V_2(\hat{s}) + 2\hat{m}_b C_7^{eff} \left[F_3(\hat{s}) + \frac{1 - \hat{M}_{K_1}^2}{\hat{s}} F_2(\hat{s}) \right] \right\} \\
D(\hat{s}) &= \frac{1}{\hat{s}} \left[\left(C_9^{eff}(\hat{s}) (1 + \hat{M}_{K_1}) V_1(\hat{s}) - (1 - \hat{M}_{K_1}) V_2(\hat{s}) - 2\hat{M}_{K_1} V_0(\hat{s}) \right) \right. \\
&\quad \left. - 2\hat{m}_b C_7^{eff} F_3(\hat{s}) \right] \\
E(\hat{s}) &= -\frac{2A(\hat{s})}{1 + \hat{M}_{K_1}} C_{10} \\
F(\hat{s}) &= \left(1 + \hat{M}_{K_1}\right) C_{10} V_1(\hat{s}) \\
G(\hat{s}) &= \frac{1}{1 + \hat{M}_{K_1}} C_{10} V_2(\hat{s}) \\
H(\hat{s}) &= \frac{1}{\hat{s}} \left[C_{10}(\hat{s}) (1 + \hat{M}_{K_1}) V_1(\hat{s}) - (1 - \hat{M}_{K_1}) V_2(\hat{s}) - 2\hat{M}_{K_1} V_0(\hat{s}) \right]. \quad (37)
\end{aligned}$$

Considering the final state lepton as muon the branching ratio for $B \rightarrow K_1 \mu^+ \mu^-$ is calculated in ref. [24] and its numerical value is

$$\mathcal{B}(B \rightarrow K_1 \mu^+ \mu^-) = 0.9_{-0.14}^{+0.11} \times 10^{-7}$$

The above value of branching ratio is for the case if one does not include $Y(\hat{s})$ in Eq. (7). The error in the value reflects the uncertainty from the form factors, and due to the variation of input parameters like CKM matrix elements, decay constant of B meson and masses as defined in Table I.

Table I : Default value of input parameters used in the calculation

m_W	80.41 GeV
m_Z	91.1867 GeV
$\sin^2\theta_W$	0.2233
m_c	1.4 GeV
$m_{b,pole}$	4.8 ± 0.2 GeV
m_t	173.8 ± 5.0 GeV
$\alpha_s(m_Z)$	0.119 ± 0.0058
f_B	(200 ± 30) MeV
$ V_{ts}^*V_{tb} $	0.0385

By including $Y(\hat{s})$ the central value of branching ratio reduces to

$$\mathcal{B}(B \rightarrow K_1\mu^+\mu^-) = 0.72 \times 10^{-7}$$

It is already mentioned that in ACD model there is no new opeartor beyond the SM and new physics will come only through the Wilson coefficients. To see this, the differential branching ratio against \hat{s} is plotted in Fig. 1 using the central values of input parameteres. One can see that their is significant enhancement in the decay rate due to KK contribution for $1/R = 200$ GeV whereas the value is shifted towards the SM at large value of $1/R$. The enhancemen is prominent in the low value of \hat{s} but such effects are obscured by the uncertainites involved in different parameters like, form factors, CKM matrix elements etc. The numerical value at these two different values of $1/R$ is

$$\begin{aligned} \mathcal{B}(B \rightarrow K_1\mu^+\mu^-) &= 0.82 \times 10^{-7} \text{ for } 1/R = 200 \text{ GeV} \\ \mathcal{B}(B \rightarrow K_1\mu^+\mu^-) &= 0.75 \times 10^{-7} \text{ for } 1/R = 500 \text{ GeV} \end{aligned}$$

The effects of UED becomes more clear if we look for the FB asymmetry in the dilepton angular distribution because it depends upon the Wilson coefficients. It is known that in SM, due to the opposite sign of the C_7 and C_9 , the forward-backward asymmetry passes from its zero position and has very weak dependence on form factors and uncertainties in input parameteres. The differential forward-backward asymmetry for $B \rightarrow K_1\mu^+\mu^-$ reads as

Figure 1:

follows[10]

$$\frac{d\mathcal{A}_{\text{FB}}}{d\hat{s}} = \frac{G_F^2 \alpha^2 m_B^5}{2^{10} \pi^5} |V_{ts}^* V_{tb}|^2 \hat{s} \hat{u}(\hat{s}) [\text{Re}(BE^*) + \text{Re}(AF^*)] \quad (38)$$

where

$$\begin{aligned} \hat{u}(\hat{s}) &= \sqrt{\lambda \left(1 - 4 \frac{\hat{m}_l^2}{\hat{s}}\right)} \\ \lambda &\equiv \lambda(1, \hat{m}_{K_1}^2, \hat{s}) \\ &= 1 + \hat{m}_{K_1}^4 + \hat{s}^2 - 2\hat{s} - 2\hat{m}_{K_1}^2(1 + \hat{s}) \end{aligned} \quad (39)$$

The variable \hat{u} corresponds to θ , the angle between the momentum of the B meson and the positively charged lepton in the dilepton c.m. system frame. In SM the zero-point of forward-backward asymmetry for $B \rightarrow K_1 \mu^+ \mu^-$ is calculated by Paracha et al. [24] and it lies at $\hat{s} = 0.16$ ($s = 4.46 \text{ GeV}^2$). They have shown that due to the uncertainties in the form factors zero position of forward-backward asymmetry A_{FB} deviate slightly from the central value in the low s region where as in the large s region these deviations are highly suppressed and zero of the forward-backward asymmetry became insensitive to these uncertainties and therefore we do not include them while analysing the above decay in UED model.

To see the new physics effects due to extra dimension, the differential forward-backward asymmetry with \hat{s} is plotted in Fig. 2. It can be seen that the zero position of forward-backward asymmetry A_{FB} shifts towards the left in ACD model with single universal extra dimension and this shifting is more clear for $1/R = 200 \text{ GeV}$. In future, when we have some data on these decays, this sensitivity of the zero position to the compactification parameter, will be used to constrain $1/R$.

The method of calculation of the form factors for $B \rightarrow K_1 \ell^+ \ell^-$ decay described in [24] can be straightforwardly used to calculate the form factors for $B \rightarrow K^* \ell^+ \ell^-$. Now after calculating these form factors for $B \rightarrow K^* \ell^+ \ell^-$ we have plotted the forward-backward asymmetry with \hat{s} in Fig. 3. We believe that it provides a useful comparison, if one compares the effect of our form factors to the zero of A_{FB} with the others like [10] and references therein. Again the zero of A_{FB} is shifted towards the left in ACD model with single universal extra dimension and this shifting is more clear for $1/R = 200 \text{ GeV}$.

Conclusion

Figure 2:

This paper deals with the study of semileptonic decay $B \rightarrow K_1 \ell^+ \ell^-$ in ACD model with single universal extra dimension which is strong contender to study physics beyond SM and has received a lot of interest in the literature. We studied the dependence of the physical observables like decay rate and zero position of forward-backward asymmetry on the inverse of compactification radius $1/R$. The value of the branching ratio is found larger than the corresponding SM value. The zero position of the FB asymmetry is very sensitive to $1/R$ and it is seen that it shifts significantly to the left. The shifting is large at $1/R = 200$ GeV and approaches to the SM value if we increase the value of $1/R$. The future experiments, where more data is expected, will put stringent constraints on the compactification radius and also give us some deep understanding of B -physics.

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Figure Captions

1): The differential branching ratio as a function of \hat{s} is plotted using the form factors defined in Eq. (31). The solid line denotes the SM result, dashed-dotted line is for $1/R = 200$ GeV and dashed line is for $1/R = 500$ GeV. All the input parameters are taken at their central values.

2): The differential forward-backward (FB) asymmetry as a function of \hat{s} is plotted using the form factors defined in Eq. (31). The solid line denotes the SM result, dashed-dotted line is for $1/R = 200$ GeV and dashed line is for $1/R = 500$ GeV. All the input parameters are taken at their central values.

3): The differential forward-backward (FB) asymmetry for $B \rightarrow K^* \ell^+ \ell^-$ as a function of \hat{s} is plotted using the form factors defined in Eq. (31) with obvious replacements for K^* . The solid line denotes the SM result, dashed line is for $1/R = 200$ GeV and long-dashed line is for $1/R = 500$ GeV. All the input parameters are taken at their central values.

4): Comparison of the differential forward-backward (FB) asymmetry for $B \rightarrow K^* l^+ l^-$ in Standard Model (SM) as a function of \hat{s} is plotted using the form factors defined in Eq.(31) vs form factors given in Colangelo *etal* [17]. The Solid line denotes the Colangelo result and dashed line denotes our result.



